Name:_____ Date:_____

SOLUTION HW Pre Calculus 12 Section 4.2 Unit Circles and Finding Coordinates of Point "P"

1. What is an Unit Circle and what does it mean to find a point on an unit circle?

   An unit circle has a radius of 1unit long. When rotating in a CCW direction, the angle of rotation is positive. In a CW direction, the angle will be negative. To find a point on an unit circle means finding a pair of coordinates on the circumference. We know that the point is correct when the sum of the squares of the coordinates is equal to 1.

2. If a point in on an unit circle, what information can you derive from the "x" and "y" coordinates?
   The square of the "x" coordinate plus the square of the "y" coordinate will be equal to 1. This is the Pythagorean thm.

3. Suppose you are told that the point "P" with coordinates (1.875, -2.45) is on a terminal rotating around the origin. Is this point on an unit circle? How would you find out?
   If $x^2 + y^2 = 1$, then the point is on the unit circle. Looking at the coordinates, we know that this is NOT true. Therefore, this point is NOT on the unit circle.

4. Suppose a point "P" on a terminal arm has rotated $\frac{5\pi}{3}$ radians. What are the coordinates of point "P" on the unit circle?

   To find the "x" coordinate take the cosine of the radian angle in radian mode. $\cos\frac{5\pi}{3} = 0.5$. To find the "y" coordinate, take the sine of the radian angle in radian mode: $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$. So the exact coordinate of point "P" when it has rotated $\frac{5\pi}{3} radians$ in standard position will be: $\left(0.5, -\frac{\sqrt{3}}{2}\right)$

5. Suppose you are given the trig ratio $\sin\theta = 0.875$, which quadrant would the angle θ be in? Explain:
   Since, $\sin\theta$ is the Y-coordinate, and the Y-coordinate is positive, the angle would then need to be in Quadrants 1 and 2. This is b/c Quadrants 1 and 2 are above the X-axis.
   The other way to do this is to use CAST. $\sin\theta$ is positive in Quadrants 1 and 2.

6. Suppose you are given the trig ratio $\cos\theta = -0.651$, which quadrant would the angle θ be in? Explain:

Since, $\cos\theta$ is the X-coordinate, and the X-coordinate is negative, the angle would then need to be in Quadrants 2 and 3. This is b/c Quadrants 2 and 3 are to the LEFt of the Y-axis.

7. Suppose $\sin\theta = m$, $\cos\theta = n$, where both "m" and "n" are negative. Then which quadrant would angle θ be in? explain:

If both the "x' and "y' coordinates are negative, then the angle would be in Quadrants 3.

8. Why is the following trigonometric ratio true? Explain: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. Is this true for all angles?

This equation is an "identity", which means that it is true for all angles. Here's the proof:

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{Opp}{Hyp}}{\frac{Adj}{Hyp}} = \frac{Opp}{Hyp} \div \frac{Adj}{Hyp}$$

$$= \frac{Opp}{Hyp} \times \frac{Hyp}{Adj} = \frac{Opp}{Adj} = \tan\theta$$

9. Suppose the point "P" is on a terminal arm that has rotated θ radians in standard position. Given that $\cos\theta = \dfrac{3}{\sqrt{11}}$, what are all the possible coordinates of point "P" on the unit circle. Provide your answer in exact form. Draw the angle θ in standard position.

i) the x-coordinate is positive. It is equal to: $\dfrac{3}{\sqrt{11}}$. Keep it in exact form.

ii) The y-coordinate can be psotive or negative. We don't know. Solve for both y-coordinates.

iii) Since it's an unit circle, we have $x^2 + y^2 = 1$. Use the Pythagorean thm to solve for "y"

$$x^2 + y^2 = 1 \rightarrow \left(\frac{3}{\sqrt{11}}\right)^2 + y^2 = 1$$

$$\frac{9}{11} + y^2 = 1$$

$$y^2 = \frac{2}{11}$$

$$y = \pm\sqrt{\frac{2}{11}}$$

Coordinates of point P are $\left(\dfrac{3}{\sqrt{11}}, \sqrt{\dfrac{2}{11}}\right) and \left(\dfrac{3}{\sqrt{11}}, -\sqrt{\dfrac{2}{11}}\right)$

10. Suppose the point "P" is on a terminal arm that has rotated θ radians in standard position. Given that $\sin\theta = \frac{-5}{3\sqrt{8}}$, what are all the possible coordinates of point "P" on the unit circle. Provide your answer in exact form. Draw the angle θ in standard position.

i) the Y-coordinate is negative. It is equal to: $\frac{-5}{3\sqrt{8}}$. Keep it in exact form.

ii) The x-coordinate can be psotive or negative. We don't know. Solve for both x-coordinates.

iii) Since it's an unit circle, we have $x^2 + y^2 = 1$. Use the Pythagorean thm to solve for "x"

$$x^2 + y^2 = 1 \rightarrow x^2 + \left(\frac{-5}{3\sqrt{8}}\right)^2 = 1$$

$$x^2 + \frac{25}{72} = 1$$

$$x^2 = \frac{47}{72}$$

$$y = \pm\frac{\sqrt{47}}{6\sqrt{2}} = \pm\frac{\sqrt{94}}{12}$$

Coordinates of point P are $\left(\frac{\sqrt{94}}{12}, \frac{-5}{3\sqrt{8}}\right) and \left(\frac{-\sqrt{94}}{12}, \frac{-5}{3\sqrt{8}}\right)$ *OR* $\left(\frac{\sqrt{94}}{12}, \frac{-5\sqrt{2}}{12}\right) and \left(\frac{-\sqrt{94}}{12}, \frac{-5\sqrt{2}}{12}\right)$

11. Suppose the point "P" is on a terminal arm that has rotated θ radians in standard position. Given that $\tan\theta = \frac{-7}{\sqrt{6}}$, what are all the possible coordinates of point "P" on the unit circle. Provide your answer in exact form. Draw the angle θ in standard position.

1. $\tan\theta$ is negative. This means, either one of the "x" coordinate or "y" coordinate is negative, and the other one is positive. So if "X" positive & "Y" negative → Q4. IF "X" negative and "Y" positive → Q2.
2. Assume $R\sin\theta = -7$ and $R\cos\theta = \sqrt{6}$. Use the Pythagorean, to find "R" and then $\sin\theta$ and $\cos\theta$

$$R^2\sin^2\theta + R^2\cos^2\theta = (-7)^2 + \left(\sqrt{6}\right)^2$$

$$R^2\left(\sin^2\theta + \cos^2\theta\right) = (49) + (6)$$

$$R^2(1) = 54$$

$$R = 3\sqrt{6}$$

3. Now go back and solve for $\sin\theta$ and $\cos\theta$. Remember, there TWO Cases:

CASE 1: Q4: "Y" coordinate is negative and "x" coordinate is positive:

$R\sin\theta = -7$ and $R\cos\theta = \sqrt{6}$

$3\sqrt{6}\sin\theta = -7$ and $3\sqrt{6}\cos\theta = \sqrt{6}$

$\sin\theta = \dfrac{-7}{3\sqrt{6}}$ $\qquad$ $\cos\theta = \dfrac{\sqrt{6}}{3\sqrt{6}}$ So the coordinates of P are $\left(\dfrac{-7\sqrt{6}}{18}, \dfrac{1}{3}\right)$

$= \dfrac{-7\sqrt{6}}{18}$ $\qquad$ $= \dfrac{1}{3}$

Case 2: Q2: The answer is more or less then same. Just that "Y" coordinate is positive and "X" coordinate is negative. Same numbers: So the coordinates of P are $\left(\dfrac{7\sqrt{6}}{18}, -\dfrac{1}{3}\right)$

12. Suppose the point "P" is on a terminal arm that has rotated θ radians in standard position. Given that $\sin\theta = \dfrac{-4}{13}$, what are all the possible coordinates of point "P" on the unit circle. What are the ratios of $\cos\theta$ and $\tan\theta$ equal to? Provide your answers in exact form. Draw the angle θ in standard position.